

# Entrance test, mathematics, 45 mins

September 10, 2010

Please clearly write your name on each sheet you use. Carefully write down each step of your solution.

1. Draw the line given by the equation  $4x + 2y = 3$ .
2. Mark all  $(x, y)$  points in the plane which satisfy the inequality  $4x + 2y \leq 3$ .
3. Solve the following system of equations:  $x - 3y = -11$  and  $2x + y = -1$ .
4. What is the length of the vector  $(4; 5)$ ?
5. Let  $a = (1; 2)$  and  $b = (5; -1)$  be two vectors. Compute and draw the vector  $a - 3b$ .
6. Simplify the fraction  $\frac{x^2-9}{x-3}$ .
7. Expand the expression  $(\sqrt{a} + \sqrt{2})^2$ .
8. Solve the quadratic equation  $x^2 + 6x - 3 = 0$ .
9. Solve the inequality  $x^2 - 5x + 6 > 0$ .
10. Solve the equation  $\sin(x) = 1/2$ . (There are infinitely many solutions.)
11. Let  $a$  be a real number. Prove that

$$4^{1/2+a/2}$$

can be simplified to  $2 \cdot 2^a$ .

12. Let  $c > 1$  be a real number. Prove that

$$\frac{c+1}{c^2-1} - \frac{c+1}{1-c}$$

can be simplified to

$$\frac{c+2}{c-1}$$